CENTRO DE INVESTIGACIÓN
E TECNOLOXÍA MATEMÁTICA
DE GALICIA

Topology Workshop – Summaries

- **Lucile Vandembrucq (Univ. do Minho),** *On the topological complexity of surfaces and other manifolds.*

  The topological complexity (TC) is a homotopy invariant which was introduced by M. Farber in order to give a topological measure of the complexity of the motion planning problem. We will review some properties of this invariant as well as its value for surfaces. We will also give sufficient conditions for the topological complexity of a closed manifold M with abelian fundamental group to be nonmaximal, that is to satisfy TC(M) < 2 dim(M). This is based on a joint work with Daniel Cohen.

- **Pablo Irimia Rega (USC),** *Geometry and Topology of Algebraic Curves.*

  The aim of this study is to prove the equality of the arithmetic and the topological genus of an algebraic projective curve. Both concepts will be introduced. Since every curve has a non-singular model, it is obtained that it can be projected isomorphically into the three-dimensional projective space. From there, it can be projected into a plane in a curve with only ordinary nodes as singularities. This curve will be a birrational model of the initial one. Given all of this, the arithmetic genus can be defined. Making use of the projection of the curve with only nodes as singularities into a line, the triangulation of the Riemann surface associated to the curve will be computed, obtaining the topological genus. This fact will lead us to obtain the equality of both genera.

- **Isaac Carcacía Campos (USC),** *Homotopic invariants in small categories.*

  In recent years there has been a constant interest in studying small categories, including examples such as: diagrams, groups, posets, groupoids… using analogous notions to those that appear when dealing with topological spaces. In this talk we will introduce some of these topological notions such as the paths in a category, the homotopy between functors, the classifying space of a category or the fibrations between categories. For this we will give some basic notions of important constructions such as simplicial sets, the interval category, the functor of geometric realization, …

- **Álvaro Carballido Costa (USC),** *An elementary proof of Hedlund's theorem.*

  Gustav A. Hedlund proved in 1936 that if S is a compact hyperbolic surface, then the horocycle flow hR over its unit tangent bundle T1S is minimal, that is, all of its orbits are dense. The goal is to give an elementary proof of this result using Furstenberg's duality as a key tool.

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[2] Hillel Furstenberg, The unique ergodicity of the horocycle flow, in Recent advances in topological dymanics (Proc. Conf., Yale Univ., New Haven, Conn., 1972; in honor of Gustav Arnold Hedlund). Springer Lecture Notes in Math., Vol. 318. , Berlin, 1973, pp. 95-115.

- **Alejandro O. Majadas Moure (USC),** *Tietze's theorem on the real line. An elementary proof.*

  We will show an elementary proof of the continuous extension theoren in the case of the real line, stated in full generality, and such that it provides an explicit and calculable expression of the extension.

- **Carlos Meniño Cotón (Univ. de Vigo),** *Leaf topology of codimension one minimal hyperbolic foliations on closed 3-manifolds.*

  A foliation by surfaces is called hyperbolic if there exists a leafwise continuous Riemannian metric of constant negative curvature. Lots of codimension one foliations on 3-manifolds are hyperbolic by Candel's Uniformization Theorem (for instance, if no invariant measure exists). It is interesting to understand where these foliations arise (examples) and how complicated the topology of its leaves can be. A foliation is called minimal if every leaf is dense in the ambient space. In this talk we deal with the question about the topology of leaves of minimal hyperbolic foliations on closed 3-manifolds, more precisely we show that every open surface can be realized as a leaf of some minimal hyperbolic foliation whose generic leaf is a plane and every open surface whose isolated ends are accumulated by genus can be realized as the leaf of some minimal hyperbolic foliation whose generic leaf is a Cantor tree or a Loch Ness Monster. Our examples impose some conditions on the ambient topology (only Seifert or graph manifolds) and transverse regularity (at most C1) and it is known that some closed 3-manifolds do not admit minimal hyperbolic foliations with rich leaf topology. This suggests an interesting interplay between leaf topology of minimal hyperbolic foliations and the ambient topology of closed 3-manifolds that is not yet well understood.

- **José M. Ribeiro Oliveira (Univ. do Minho),** *Couplings on Lie groupoids and Lie algebroids*

  A coupling of a Lie algebroid A with a Lie algebra bundle K is morphism from A to the Lie algebroid of the outer derivations of K. After reviewing general definitions of higher Lie theory and some relations with foliations, we will mention that couplings may not exist and will propose conditions for a coupling to exist when the bases are foliated spaces.

- **Reihaneh Mehrabi (USC),** *Mayer-Vietoris sequence for generating families in diffeological spaces.*

  We prove a version of the Mayer-Vietoris sequence for De Rham differential forms in diffeological spaces. It is based on the notion of a generating family instead of that of a covering by open subsets.

- **Alba Sendón Blanco,** *A (very introductory) introduction to intersection (co)homology.*

  Poincaré duality is one of the most famous mathematical theorems: if we have a closed, connected, oriented manifold, then the cap-product with the fundamental class induces an isomorphism between its homology and cohomology of complementary dimensions. This property fails for more general spaces. M. Goresky and R. MacPherson's intersection homology, based on a notion of perversity, restores Poincaré duality for some singular spaces such as pseudomanifolds. For obtaining a Poincaré isomorphism in this scenario, we need an intersection cohomology theory. The first candidate comes from the linear dual, and a more elaborated option is based on a simplicial blowup. During the talk, we will explain intersection homology and both intersection cohomology theories, seeing some duality results already achieved with each of the choices.

- **Pablo Montaña Fernández (USC),** *Dimension of algebraic varieties.*

  Algebraic varieties are defined as the set of points satisfying a system of polynomial equations. They are the closed sets in a topology: the Zariski topology. The lack of a result akin to the inverse function theorem implies that the notion of dimension concerning this kind of sets demands the introduction of notions of dimension within the context of commutative rings.

- **Samuel Castelo Mourelle (USC),** *The Lefschetz fixed point theorem.*

  The goal of the talk will be the Lefschetz fixed point theorem as well as some of its applications. We will work mostly with a simplicial vision in mind. Therefore, we begin by introducing the concepts related to simplices and simplicial complexes needed to build the simplicial homology. Then, once we have all the basic definitions, we will state the theorem, giving a sketch of the proof. Finally, we will see that, even though formally speaking it is a theorem in the setting of Algebraic Topology, its applications go way beyond this field of mathematics, like Algebra, Geometry or Analysis, being this one the most obvious as it is a fixed-point theorem. In addition, some comments about the posets version of the theorem will be made as well.